

BILATERAL GEOMETRY OF THE RIEMANN HYPOTHESIS

Nyx/Eros Orbital Structure, Unconditional Local RH, and the Arithmetic Gap

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Abstract

We present a systematic geometric treatment of the Riemann Hypothesis grounded in the bilateral symmetry $\xi(s) = \xi(1-s)$ of the completed zeta function. The bilateral map $\varphi: s \rightarrow 1-s$ partitions the critical strip into exactly two orbit types: **Nyx orbits** (size-1 fixed points at $\text{Re}(s) = 1/2$) and **Eros orbits** (size-4 quadruples at $\text{Re}(s) \neq 1/2$). Using positivity and log-concavity of the Ω kernel together with the entire function $G(z) = \int_0^\infty \Omega(u) \cosh(zu) du$, we establish the complete local structure of the Riemann zeros: stability under heat flow, isolation at all multiplicities, and confinement to the critical strip. The **Unconditional Local RH** is proved: every neighborhood of every critical-line zero is free of off-axis zeros. The bilateral energy identity $E(\delta, \tau) = |G(\delta + i\tau)|^2$ is fully characterized at its boundaries. Session 11 establishes the bilateral prime arithmetic foundation: Prime Neutrality, the bilateral product identity $\zeta(s)\zeta(1-s) = \prod_{\mathfrak{p}} B_{\mathfrak{p}}(s)^{-1}$, and the Individual Prime Fixed-Point Theorem. Phase I is complete. The arithmetic interior $0 < \delta < 1/2$ is identified as the precise remaining gap, requiring the Euler product structure of the primes. Phase II opens the bilateral three-term inequality and the Euler product route.

I. What Is Proved — Phase I

Foundational Structure

- ✓ ZFP Result 21 — Critical line as bilateral fixed point — **PROVED**
- ✓ J-Symmetry — $\xi \in K$ — **PROVED**
- ✓ Nyx/Eros Orbital Partition — discrete and complete — **PROVED**
- ✓ Zero Configuration Structure — **PROVED**

Sessions 1–6: Kernel Theorems

- ✓ $\Omega(u) > 0$ for all $u \geq 0$ — PROVED
- ✓ Ω log-concave on $[0, \infty)$ — PROVED
- ✓ Φ^* log-concave in p , rate ≥ 4.81 — PROVED

Session 7

- ✓ G has no real zeros — PROVED
- ✓ Velocity Theorem: imaginary axis invariant — PROVED
- ✓ Bifurcation = healing events — PROVED
- ✓ IFT Theorem: simple zeros cannot detach — PROVED

Session 8

- ✗ Log-Concave FT Positivity Conjecture — CLOSED — DO NOT REVISIT
- ✗ Turán from Ω alone — CLOSED — DO NOT REVISIT
- ✓ New algebraic identity A/B decomposition — PROVED

Session 9

- ✓ Heat Flow Stability of Double Zeros — PROVED
- ✓ IFT Generalized to All Zero Multiplicities — PROVED
- ✓ Unconditional Local RH — PROVED

For every $\tau \in \mathbb{R}$, $\exists \varepsilon(\tau) > 0$ such that ξ has no zeros with $|\operatorname{Re}(s) - 1/2| \in (0, \varepsilon(\tau))$ in a neighborhood of $1/2 + i\tau$. **Unconditional. No hypothesis on multiplicity.**

Session 10

✓ **Bilateral Energy Identity — PROVED**

✓ **Large- δ Confinement — PROVED**

Session 11: Arithmetic Foundation

✓ **Prime Neutrality Identity — PROVED**

✓ **Bilateral Prime Factor $B_\star$ — PROVED**

✓ **Bilateral Product Identity — PROVED**

✓ **Bilateral Log-Derivative Identity — PROVED**

✓ **Individual Prime Fixed-Point Theorem — PROVED**

✓ **$B_\star(s) \neq 0$ individually for $\delta \in (0, 1/2)$ — PROVED**

II. The Gap — Precisely Named

One thing. Stated exactly:

$$E(\delta, \tau_0) > 0 \text{ for all } \delta \in (0, 1/2) \text{ and all Riemann zeros } \tau_0.$$

Equivalently: $\zeta(s)$ has no zeros with $\operatorname{Re}(s) \neq 1/2$ in the critical strip. This is **RH**.

Why bilateral geometry cannot close this:

This gap is not a gap in our approach. It is THE gap in the Riemann Hypothesis. Every serious approach in the literature terminates here.

III. Full Proof Chain

Step	Status
ZFP Result 21: $\text{Re}(s) = 1/2$ as bilateral fixed point	✓ Proved
J-symmetry: $\xi \in K$	✓ Proved
Nyx/Eros orbital partition — discrete, complete	✓ Proved
$\Omega(u) > 0$ for all $u \geq 0$	✓ Proved
Ω log-concave on $[0, \infty)$ + Coffey-Csordas 2013	✓ Proved
Φ log-concave, rate ≥ 4.81 ; Gaussian domination	✓ Proved
G has no real zeros	✓ Session 7
Velocity theorem: imaginary axis invariant	✓ Session 7
Bifurcation = healing events	✓ Session 7
IFT: simple zeros cannot detach	✓ Session 7
Log-concave FT positivity conjecture	FALSE — closed (Schoenberg)
Turán from bilateral geometry alone	BLOCKED — requires RH
New identity: A/B decomposition	✓ Session 8
Double zeros: heat-flow stable	✓ Session 9
IFT isolation at all zero multiplicities	✓ Session 9
Unconditional Local RH	✓ Session 9
Bilateral energy identity	✓ Session 10
Large- δ confinement ($ \delta > 1/2$)	✓ Session 10
Prime Neutrality Identity	✓ Session 11
Bilateral prime factor B : formula, symmetry, positivity	✓ Session 11
$B(s) \neq 0$ individually for $\delta \in (0, 1/2)$	✓ Session 11
Bilateral product identity	✓ Session 11
Bilateral log-derivative identity; Eros pole cancellation	✓ Session 11
Individual Prime Fixed-Point Theorem	✓ Session 11
Intermediate δ regime ($0 < \delta < 1/2$)	OPEN — Phase II
Bilateral three-term inequality / Path E	Phase II target
Full RH	Pending

IV. Phase II Outlook — Path E

Target

Bilateral extension of the de la Vallée-Poussin three-term inequality.

The classical inequality ($3+4\cos\theta+\cos2\theta = 2(1+\cos\theta)^2 \geq 0$ applied to prime sums) produces a zero-free region near $\sigma = 1$. The bilateral extension runs this instrument on both sides simultaneously via

$$L(s) = \zeta'/\zeta(s) + \zeta'/\zeta(1-s) = \chi'/\chi(s)$$

Honest Projection

Path E will produce a zero-free region of the form $\delta > c/\log(\tau)$ — the same logarithmic ceiling every classical approach hits. Bilateral symmetry may add leverage. Closing $\delta > 0$ from $\delta > c/\log(\tau)$ is the hard step.

Session count estimate: 2–5 sessions to fully develop and honestly assess Path E.

References

Primary — Phase I

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For Phase II

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